Non-equilibrium dynamics in urban traffic networks

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Urban traffic is a complex phenomenon, it has emergent patterns that we can observe and quantify in the streets. Non equilibrium relations between the flow of cars and their spatial density have successfully informed traffic models over the years. However, at urban scale, characterizing the transition that gives rise to the appearance and growth of congestion bottlenecks remains an elusive task. The connection between streets topology and the spatial dynamics of the population are still a missing component to comprehensively describe the emergence of urban congestion ubiquitous across the world. Informed by the trips of millions of mobile phone users in the morning peak hour from five diverse cities, we show that the level of congestion can be characterized by a time ($\tau$). That is the characteristic time that takes to the entire network to decrease the number of vehicles after it reaches its maximum at the peak hour. It depends on two quantities that can be measured in cities worldwide. The first is the distribution of free flow travel times ($t_{ff}$) given by trip distances, and the second is the road usage ($\Gamma$), given by the total vehicle demand divided by the road supply. Changing either of these two quantities to improve $\tau$, can directly inform interventions to improve congestion in any city. Moreover, if we increase the volume of cars in the network, keeping the
road capacity and the empirical spatial dynamics from origins to destinations unchanged, we identify a distinctive phase transition to a collapsed state. For the first time, the transition to urban gridlock is defined in terms of an unloading time or recovery period from real travel demand. We show that it has characteristics of a non-equilibrium phase transition analogous to directed percolation.

1 Introduction

Traffic jams have been successfully studied through the lens of statistical physics. The availability of camera data have allowed to model the dynamics of jams in the freeways. The relation between flow of cars in the roads and their density is described via the well established fundamental diagram. The cars travel in free flow and change their state to a jammed state that moves through the roadways. Congestion in a road emerges as spontaneous transitions between these two phases.

In turn, congestion at urban scale is the result of the interplay between the travel distribution of vehicles and the physical constraints of the roads, such as: network geometry, road lengths, road capacity and speed limits. This leads to saturation of most frequently used road segments spilling over into nearby streets, and large traffic jams or urban gridlocks emerge. In extreme cases these jams can sometimes collapse the entire system and even last for days to resolve.

At the scale of road networks, the transition to congestion has mainly been studied by simulations. Currently, there are two accepted approaches. The first consists in an agent-based...
model inspired by the traffic dynamics of the Internet \textsuperscript{11,12}. The increase of volume demand $R$ in the network beyond a threshold, induces a transition to a congested state, where the number of vehicles in the network starts increasing steadily in time, and the number of cars that accumulates per unit time defines the order parameter. The aim of this approach consists of identifying the critical value $R_c$ at which vehicles start accumulating in the network and understanding how the various elements of the model (network topology, road capacity and routing strategy) affect the transition to a global congested state. The second approach relies on the existence of a well-defined relationship between network-wide average flow and the density of cars. The traffic dynamics is modeled via dynamic traffic assignment \textsuperscript{13}, as well as macroscopic calculations based on car-following or cellular automata models \textsuperscript{14–17}. In these cases, variations in flow and density can be measured at the road segment level. Considering the network density as the control parameter, similarly to the single road case, a congested state at the network level is defined as a drop in the cars out-flow when the density exceeds certain threshold value. More recently, a study \textsuperscript{18} based on taxi data that measured their speeds on the streets, revealed that changes in the state of the road network can be described by a percolation process where the number of road links below a certain speed threshold form a giant component in the rush hour.

Until now, the lack of data for travel demand has prevented large-scale comparative studies of the relation between the empirical urban congestion and global traffic parameters such as car volume, road network supply and distances traveled. Namely, there are two limitations in the simulation-based studies. One is the use of random origin-destination tables (OD) that neglects the inherent spatial heterogeneity in the demand distribution. The other is the interpretation of
transition to congestion as a loading process, where the order parameter is defined by the amount
of accumulated vehicles. These limitations generate two main consequences. First, the transition
to congestion occurs at unrealistic high car densities. Second, it does not have relation to the
empirical travel demand, which creates gridlocks in most used streets. Recently, Mahmassani et
al.\(^{13}\) characterize urban gridlock using descriptive measures from simulations of the Chicago road
network. This work pioneered the concept of the recovery time as an important metric to diagnose
urban congested states. The recovery time is the time vehicles take to reach to their destination after
their maximum accumulation in the rush hour. Yet its connection with other global characteristics
of urban travel remained missing.

The recent availability of data on personal tracking devices allows us to overcome the named
limitations. Travel information can be extracted from the analysis of call detailed records (CDRs)\(^{19-22}\)
from mobile phones. Çolak et al.\(^{23}\) have estimated the origin-destination tables (ODs) during
the morning peak hour for five major cities around the world: Boston, San Francisco Bay, Rio de
Janeiro, Lisbon and Porto. Using these ODs, along with road networks publicly available on Open-
StreetMaps the congested travel time \(t_{ue}\) for each road was estimated and at the network scale
and the ratio of road supply to vehicles demand \(\Gamma\) explained the differences in the percentage of
congestion reported in empirical GPS data from each of the five cities.

In this work we use the travel demand information in the same cities as an input to a cel-
lular automata microsimulation\(^{24}\), to study the morning peak hours as a loading and unloading
process\(^ {13}\). We find that under current traffic conditions, the congestion state of the city is defined
by the recovery time $\tau$. Interestingly, $\tau$ can be estimated from two quantities that are measurable properties of each city. The first, is the distribution of trip distances that gives us the median free flow travel time on the network, $t_{ff}$. The second is $\Gamma$, defined by the ratio of the total travel demand in the roads and the total road supply. Moreover, we do scenario analysis and show that by increasing the volume of cars in the network, a critical transition to a network congested state emerges. We characterize this transition and discuss its non-equilibrium nature.

2 Method

We start from car OD volumes from 7:00 to 8:00. The trip in the road networks is pre-calculated with the congested traveled time $t_{ue}$ as weights. We assign initial routes to each vehicle by means of shortest time path. Vehicles are created at origin nodes (intersections) and then inserted in the network according to the initial routing strategy, such that, for high insertion rates, queues of new vehicles can be formed at origin nodes. Vehicle dynamics along road segments is modeled with the deterministic Nagel-Schreckenberg CA model \(^{24}\). When a car is traveling in free flow and approaches an intersection it decreases its speed to $v_{uts}$. In the implementation presented here our unit time step (uts) is $\Delta t = 1.2375[s]$ and the cell sizes are $l=5.5[m]$. This defines the minimum distance per unit time step that vehicles can travel in the absence of congestion, which corresponds to $v_{uts} = 16[km/h]$. Each vehicle keeps a gap distance which creates congestion when cars accumulate in the streets taking speeds in the range $[0, v_{max}]$, $v_{max}$ is taken from the speed limit of the road network data (see Fig 1).
The dynamic at the intersections has three steps. First, the first *new* vehicle in the queue is chosen to pass. Secondly, incoming streets are checked in a random sequence asking for the destination of the first vehicle in the street. If the intersection is, just the destination node, then the vehicle is removed from the network. Otherwise, if the first cell of the desired destination street is free, the vehicle is delivered with a probability \( p \) proportional to the empirical flow capacity of the originating street, \( C_e \) (see Supplementary Table 1 for the values used). Consequently, here is where the bottlenecks generate, due to capacity limitations and spill back effects. Finally, in case of long waiting times, we introduce a basic dynamic routing strategy: A vehicle that has been stopped at an intersection during more than \( t_{\text{wait}} = 96 \) time steps (approx. 2 minutes) can decide to reroute to a less congested destination street and recomputes its route. Under low traffic conditions, this strategy softens the artifacts that can be generated by short-length cycles, e.g. roundabouts intersections, where the turning maneuvers can create unsolvable gridlocks. In high traffic cases, the possibility of re-routing helps to solve gridlocks, and to reach a recovery period for congestion. The values of speed limit and flow capacity are empirical variables provided by the road network data.

Rush hours can be described as a a loading and unloading process. Thus, to model the morning peak hours, we load the road network during one hour; i.e. every \( \Delta t \), the network is loaded with \( R \) random chosen trips. After that, we stop the loading and let the system recover within a time window of 9 hours (see Fig 2). We select a sufficiently long time of observation that allows us later to observe the dynamics of long-lasting traffic jams.
3 Current Traffic Conditions

First, we focus on understanding the system behavior under actual traffic conditions. To simplify the cellular automata simulations, we use here a one-lane representation of the streets. In consequence, the total volume demand ($V$) is re-scaled by the ratio between the space demands in simulated and real networks, i.e.

$$V_{\text{now}} = \frac{\sum_{e \in E} x_e \cdot \ell_e}{\sum_{e \in E} x_e \cdot \ell_e \cdot n_e} \cdot V,$$  \hspace{1cm} (1)

where $x_e$, $\ell_e$ and $n_e$ are the vehicle flow, length (in km) and number of lanes of a road segment $e$.

Despite the differences in road infrastructure, there are slight differences between these ratios for the considered cities, as seen in Table 1. This is just further evidence of how traffic streams are concentrated on a few arterial roads only. Next, we distribute the re-scaled volume demand ($V_{\text{now}}$) in the one hour loading, such that $R_{\text{now}} = \frac{V_{\text{now}}}{1h} \cdot \Delta t$ random chosen trips are inserted each time step $\Delta t$.

Figure 2a shows the schematic representation of the percentage of vehicles in the network, $n(t) = \frac{N(t)}{V_{\text{total}}}$, where $V_{\text{total}}$ is the total number of vehicles to be inserted. We can see that the network responds differently to loading and unloading, showing hysteresis effects, as other studies have also uncovered $^{13,25,26}$. Interestingly, for all five cities the recovery period follows an exponential decay,

$$n(t)_{t \geq 1h} = n(1h)e^{-\frac{t-1h}{\tau}},$$  \hspace{1cm} (2)

with the unloading time $\tau$ ranging from 0.49 to 1.26 hours (see inset Fig. 2a). This behavior implies
a proportionality between the outflow rate and the number of vehicles in the network,

\[
\frac{dN(t)}{dt} = -\tau \cdot N(t),
\]

which in fact is confirmed by simulations, as seen in Fig. 5b (see also Supplementary Fig. 1). We are interested in understanding what characterizes this exponential recovery. Therefore, we begin by analyzing the travel demand. Similar to Ref. 23, commuting trip distances, \(d\), can be approximated by a log-normal distribution (see Supplementary Fig. 2),

\[
P(d; \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma d} e^{-\frac{(\ln(d) - \mu)^2}{2\sigma^2}}
\]

with medians ranging from 10 to 16 [km] and the s.d. ranging from 2.13 to 2.91 km: \(\mu(\sigma)_{\text{Boston}} = 2.31(1.07)\), \(\mu(\sigma)_{\text{Porto}} = 2.52(0.76)\), \(\mu(\sigma)_{\text{Lisbon}} = 2.77(0.91)\), \(\mu(\sigma)_{\text{Rio}} = 2.48(1.04)\) and \(\mu(\sigma)_{\text{SF Bay}} = 2.73(0.97)\). After the 1st hour there is an important fraction of vehicles that have not reached their destination. The remaining distance of these vehicles follows a Weibull distribution, \(P(x; \kappa, \lambda) = \frac{\kappa}{\lambda} \left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-\left(\frac{x}{\lambda}\right)^\kappa}\)

(as presented in Fig. 2c left). In a non-collapsed state, the remaining travel times, \(\hat{t}\), follow the same distribution, as shown in Fig. 2c right. Thus, in the recovery period after the first hour, the number of vehicles as a function of time, \(t_r = t - 1h\), can be estimated as

\[
N(t_r) = N(1h) \cdot \left(1 - \int_0^{t_r} P(\hat{t}; \kappa, \lambda) \, d\hat{t}\right) = N(1h) \cdot e^{-\left(t_r/\lambda\right)^\kappa}
\]

As \(\kappa\) takes values very close to 1.0 and the \(\lambda\) values are similar to the \(\tau\) of each city, this explains the exponential recovery time.

The network response to the traffic demand can be measured by how long it takes to the network to be unloaded. We argue that \(\tau\) quantifies the congestion level of the road network. With this in mind, we now focus on how the travel demand and network features explain \(\tau\). To that end,
we measure two characteristics for each city. The first is the demand-to-supply ratio defined as
\[ \Gamma = \frac{\sum_{e \in E} \ell_e x_e}{\sum_{x_e > 0, e \in E} \ell_e C_e}, \]  
(5)
which is a dimensionless metric that captures the spatial distribution of the loading on the available road infrastructure of the city. The second, is the median of the free flow travel time \( t_{ff} \), defined as the travel time at the speed limit over all the trips in the rush hour, if they occurred without congestion. This provides a time dimension and captures a feature of the travel demand related to the form of the city and the distribution of residential places respect to the required destinations. The values of both parameters are shown in Table 2. As shown in Fig. 2d there is a clear linear relation between \( \tau \) and the product \( \Gamma \times t_{ff} \), such that the congestion level of each city can be defined only by these two observable that synthesize travel demand, road infrastructure and road usage characteristics.

4 Phase Transition to Urban Gridlock

While the considered cities already face a high traffic demand, the studied exponential decay indicates a rapid recovery without the occurrence of long-lasting traffic jams, and still the minority of the available streets are in highly congested state. We further study controlled increments of the cars volume, while maintaining the same spatial distribution of the ODs and the street capacity.

Figure 3a shows the recovery period for all five cities under different loading rates. As \( R \) increases, the initially exponential unloading becomes a algebraic decay (\( \sim t^{-\alpha} \)) beyond a critical value \( R_c \). For \( R > R_c \), the unloading follows a slower recovery until an inflection point appears,
where the outflow rate decreases to very low levels and remains so for a considerable time. For example, the outflow for a rate greater than $R_c$ is shown in Boston in the plot of Fig. 5a, and the inflection point occurs after 4 hours. Eventually, due to the rerouting possibilities, the system returns to its normal unloading (not shown in the figure). By defining the percentage of vehicles in the network $n(t)$ as the order parameter, the dynamics of the system resembles the critical behavior of the directed percolation (DP) universality class, with the difference that traffic systems do not have irreversible absorbing states due to the rerouting rules.

From the slopes of the algebraic regimes for the two R values (lines marked with symbols) closest to the transition threshold, we estimate the critical loading rate $R_c$, and the critical exponent $\alpha$. Furthermore, we follow the scaling approach in the DP framework [27–29], and the curves collapse when $n(t) \cdot t^\alpha$ is plotted as a function of $(t - 1)|\varepsilon|$, where $\varepsilon = \frac{R^2 - R_c^2}{R_c^2}$ is the deviation from criticality, see Fig. 3b. Even though $\alpha$ is different for each city, indicating non-universality, this behavior evidences that the onset of the traffic gridlock is a non-equilibrium phase transition [27–29].

We further study the transition using the loading rate as control parameter. We define the remaining percentage of vehicles in the network at long times $n(t >> 1 + \tau)$ as an order parameter. We arbitrarily test here $t=1h + 7\tau$ which is always after the inflection point. Note this is a factor of the current recovery time ($\tau$), allowing us to compare cities and the magnitude of traffic collapse with respect to their current state. Figure 4a shows $n(1h + 7\tau)$ as a function of $\frac{R}{R_c}$ for all cities. As depicted, with increasing $\frac{R}{R_c}$, $n(1h + 7\tau)$ increases continuously at the vicinity of $\frac{R}{R_c}$=1.0. For this transition of second-order, the variation of $n(t_o + 7\tau)$ follows a power law, $n(t_o + 7\tau) \sim \varepsilon^\beta$
shown in the inset of Fig. 4a. Figure 4b shows how the \( \beta \) increases with the increasing of \( \alpha \).

Remarkably, the values of \( \beta \) range in between \( \beta_{1D} \) and \( \beta_{2D} \), the universal exponents of (1+1) and (2+1)-dimensions in DP. We interpret this behavior as a dimensional crossover from 1D to 2D, where respectively Porto and Boston are close to each limit, going from one effective dimension to the other as traffic performance is improved. Boston has a more filled space with grid-like streets and Porto has more empty zones.

We define a third order parameter associated with the temporal dynamic. First, we identify the value of \( n(\tau) \) for the empirical loading rate \( R_{now} \) in cities today (see Fig. 2a). We then measure the recovery time \( T \), defined as how long it takes to reach the current value of \( n(\tau) \), for \( R > R_{now} \).

The definition of \( T \) is illustrated in Fig. 2. In order to compare the response of various cities, we define \( \left( \frac{T}{\tau} \right)^{\alpha} \) as the temporal order parameter. As expected, Fig. 4B shows that this quantity diverges at the vicinity of \( \frac{R}{R_c} = 1.0 \).

Figure 5a depicts the temporal evolution of the Boston road network for \( R > R_c \). After the first hour, the system reaches a high occupancy with some traffic jams formed around low-capacity configurations as roundabouts and intersections of two-way streets. Then, during a slow and short recovery period, vehicles with reasonably congested routes can reach their destinations. However, most of the vehicles remain stuck in large traffic jams that become long-lasting gridlocks. Finally, only when these gridlocks are resolved, the system return to its normal unloading. In analogy to the DP framework, the emergence of long-lasting gridlocks play the role of the reversible absorbing states which can be solved due to the re-routing behavior. Fig. 5b shows that, under current
traffic conditions, the rate of outflow changes after the peak time and that the exponential decay characterized by $\tau$ occurs only in the unloading phase. Yet, we have demonstrated that much information of the dynamics of urban traffic can be captured in the unloading phase.

5 Discussion

We have studied the transition to congestion in urban road networks informed by empirical travel demand and road capacity. The transition is defined in terms of an unloading time or recovery period that has analogies with a non-equilibrium phase transition and resembles directed percolation. We focus on analyzing the urban traffic congestion as a loading and unloading process. From simulations of the morning peak hours, we show that the congestion level of the five subject cities can be described by two observable quantities, car demand to road supply ratio and the median of the travel time distribution in free flow. Our results offer quantitative insights of the interplay in urban traffic dynamics between the available capacity of road infrastructure and the travel demand defined by both the distances traveled and the number of cars. Furthermore, we study the emergence of congestion increasing the number of cars, keeping the trip distributions and street capacities unchanged. In this case, we show that the transition to urban gridlock, resembles the directed percolation universality class, and it can be studied as a non-equilibrium phase transition. Future challenges include further unveiling the nature of the critical behavior and measuring the clustering dynamic of the congested links. This work uncovers a novel relevance on the dynamics of the recovery period and relates it to measurable urban quantities based on actual travel demand. In a new data rich reality, we suggest a change of paradigm, opening avenues for the physics of
urban traffic.
Table 1: A comparison of the travel demand and road network in the subject cities

<table>
<thead>
<tr>
<th>City</th>
<th>Boston</th>
<th>Porto</th>
<th>Lisbon</th>
<th>Rio</th>
<th>SF Bay</th>
</tr>
</thead>
<tbody>
<tr>
<td>roads (th. miles)</td>
<td>12</td>
<td>3</td>
<td>7</td>
<td>6</td>
<td>30</td>
</tr>
<tr>
<td>total volume demand, (V) (mil.)</td>
<td>0.916</td>
<td>0.171</td>
<td>0.324</td>
<td>0.432</td>
<td>1.015</td>
</tr>
<tr>
<td>(V_{now}/V)</td>
<td>0.417</td>
<td>0.403</td>
<td>0.414</td>
<td>0.439</td>
<td>0.391</td>
</tr>
<tr>
<td>(R_{now}[veh/\Delta t])</td>
<td>131</td>
<td>24</td>
<td>48</td>
<td>64</td>
<td>134</td>
</tr>
</tbody>
</table>

Table 2: Comparison of metrics associated with urban traffic in the subject cities

<table>
<thead>
<tr>
<th>City</th>
<th>Boston</th>
<th>Porto</th>
<th>Lisbon</th>
<th>Rio</th>
<th>SF Bay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demand-to-supply, (\Gamma)</td>
<td>0.129</td>
<td>0.101</td>
<td>0.121</td>
<td>0.180</td>
<td>0.213</td>
</tr>
<tr>
<td>Mean commuting dist. (d[km])</td>
<td>10.07</td>
<td>12.45</td>
<td>15.99</td>
<td>11.98</td>
<td>15.33</td>
</tr>
<tr>
<td>s.d. commuting dist. ([km])</td>
<td>2.91</td>
<td>2.13</td>
<td>2.48</td>
<td>2.82</td>
<td>2.64</td>
</tr>
<tr>
<td>Mean free-travel-time, (t_{ff}[h])</td>
<td>0.184</td>
<td>0.188</td>
<td>0.267</td>
<td>0.218</td>
<td>0.234</td>
</tr>
<tr>
<td>Mean remaining dist. (\hat{d}[km])</td>
<td>17.69</td>
<td>12.69</td>
<td>20.47</td>
<td>18.44</td>
<td>24.00</td>
</tr>
<tr>
<td>Mean remaining travel time, (\hat{t}[h])</td>
<td>0.496</td>
<td>0.534</td>
<td>0.843</td>
<td>1.094</td>
<td>1.177</td>
</tr>
</tbody>
</table>
References


26. V. Gayah and C. Daganzo, “Clockwise hysteresis loops in the macroscopic fundamental di-


Figure 1: Model dynamics. Once vehicles are inserted in the network, the trip toward their destinations are determined by two process: the dynamics along the streets and the routing at the intersections. (Right) Vehicle dynamics along road segments is modeled with the deterministic Nagel-Schreckenberg CA model. For simplicity, each edge only has one lane. Therefore, every road segment is discretized in cells of equal length, $l=5.5\,[m]$, and no more than one vehicle can occupy a cell at every time step $\Delta t=1.2375\,[s]$; hence the speed unity is fixed to $v_{uts}=16\,[km/h]$ corresponding to 1 cell per time step. At each time step, all vehicles update in parallel their velocities and move according to the rule $v_{t+1}=\min(v_t + 1, gap, v_{\text{max}})$ where $gap$ is the distance from the car ahead or to the next intersection. The snapshot illustrates the various cases of the dynamics, black vehicles correspond to the configuration at time $t$ and grey ones correspond to the position at $t+1$ after the velocity updating. (Top) Each timestep in a random sequence, the intersections transmit the first vehicles of the originating streets. The vehicles can be routed (1) if the first cell in the desired destination street is empty and (2) if the road capacity of the originating street allows it. In the later case, the vehicle is delivered with a probability $p$ proportional to the road capacity of the originating street, $C_e$. In case of long waiting times, vehicles re-route. In the snapshot colored arrows illustrate several possibilities of moving: green (successful), red (not successful) and blue (a new route chosen). A light-blue vehicle means that the cell is occupied by a vehicle that has crossed before or by one that just entered the network.
Figure 2: **Comparison of congestion levels for five studied cities.** (a) Schematic curve of the fraction of vehicles in the network, $n(t) = \frac{N(t)}{V_{total}}$, for simulations of the morning peak hour. (b) During the 1st hour every city is loaded homogenously with a rescaled loading rate $R = R_{now}$ (see Table 1) from their empirical travel demand. The inset shows the log-lin plot of the recovery period with the unloading time $\tau$ of the fitted exponential function depicted in the legend. Every curve here is an average over 20 realizations. (c) After the loading hour, the remaining distance $\hat{d}$ and the remaining travel time $\hat{t}$ for the cars in the network can be fitted by a Weibull distribution with parameters shown in the legends (see Table 2). This explains the exponential behavior in (b). (d) Linear relation between $\tau$ and $\Gamma \times t_{ff}$, the product of the demand to supply ratio and the free travel time of the median distance traveled for each city. Error bars show the s.d. Remarkably, we also find a linear relationship between $\tau$ and the TomTom Traffic Index reported in 2016, as shown in Supplementary Fig.3.
Figure 3: Critical behavior of the fraction of vehicles in the network $n(t)$. To study the transition to urban gridlocks, we systematically increase the car demand per time step on each city. (a) Number of vehicles in the network $n(t)$ after the loading, for several loading rates. Critical condition is indicated by the solid black lines where the $n(t)$, the order parameter, follows a power law $t^{-\alpha}$. The non-universal values of the critical exponent $\alpha$ are shown for each city. The critical loading rate $R_c$ is estimated as the average loading rate of the two colored curves. (b) Scaling plot of data in (a), where $\varepsilon = \frac{|R^2 - R^2_c|}{R^2}$ is the deviation from criticality. Above the critical point $R > R_c$, the system falls into a gridlock state, that eventually resolves after several hours.
Figure 4: **Phase transition vs. the control parameter** $R/R_c$. (a) Fraction of vehicles at $t=1h+7\tau$ vs $R/R_c$ for the five cities. Inset: same data in logarithmic scales, the solid lines show $\varepsilon^\beta$. (b) Relationship between the critical exponents $\alpha$ and $\beta$. Note that the black lines (in (a) and (b)) depict the universal values of $\beta_1$ (dashed) and $\beta_2$ (solid) of directed percolation. (c) Recovery time $T$ diverges at the critical point. A data collapse can be obtained by plotting the quantity $(T/\tau)^\alpha$. 
Figure 5: **Outflow rate.** (a) Snapshots of the Boston network at three different times of its evolution for $R>R_c$. We consider the normalized speed on every link, that is, the current speed (with 4-minutes resolution) divided by the speed limit of the street. Thus, the road segments are classified into three categories according to the speed ratio: below 0.5 (red), between 0.4-0.9 (yellow) and above 0.9 (green). Observing the clusters of each color, one can realize that at $t=1h+\tau$, giant traffic jams are already formed. At $t=1h+7\tau$, most vehicles are trapped in the gridlock. Finally at $t=1h+14\tau$ (≈ 8.5 hours), the long-lasting traffic jams are still being resolved. The plot show the outflow of vehicles vs time (for the other cities see Supplementary Fig. 4). (b) Relationship between network outflow rate $\frac{dN(t)}{dt}$ and the number of vehicles in the network $N(t)$ for three of the cities (for the rest see Supplementary Fig. 1). Shortly after finishing the loading, all cities exhibit a linear relationship in the unloading phase. The clockwise hysteresis loop is wider as the congestion level in the city is larger.